Field Equations for Localized Individual Photons and Relativistic Field Equations for Localized Moving Massive Particles

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Abstract :

Calculation of the energy of localized electromagnetic particles by integration of energy fields mathematically deemed spherically isotropic and whose density is radialy decreasing from a lower limit of $\lambda\alpha/2\pi$ to an infinite upper limit (∞), allowing the definition of discrete local electromagnetic fields coherent with permanently localized moving particles.

Extended Abstract :

When localized electromagnetic particles are considered, the only way ever devised to sum up by integration their total complement of energy, which is deemed to be spherically isotropic and mathematically deemed to radialy decrease to infinity, involves setting the upper limit of integration to infinity, and setting the lower limit to a specific distance from zero simply because integrating up to the center of the particle ($r = 0$) would integrate an infinite amount of energy.

Using this established method, and quantizing the unit charge in the Biot-Savart equation, physicist Paul Marmet [1] established an equation allowing calculating the total relativistic mass of the magnetic field of a moving electron, from which can be deduced the invariant mass of the magnetic field of an electron at rest. The lower limit of integration in the case of an electron turns out to be the electron Classical Radius ($r = 2.817940285\text{E-}15$ m).

From working on other aspects of electromagnetic theory [3], I had previously come across the fact that the classical radius of the electron was obtained by multiplying the amplitude of the electron Compton wavelength by the fine structure constant ($r_e=\lambda_c\alpha/2\pi=2.817940285\text{E-}15$ m) and that the Compton wavelength itself was the actual absolute wavelength of the energy making up the rest mass of the electron ($\lambda_c = h/m_0c=2.426310215\text{E-}12$ m).

This led me to consider the possibility that the total complement of energy of any localized electromagnetic particle could possibly be obtained by integrating their energy in the same manner, that is by setting the upper limit of integration to infinity, of course, and by setting the lower limit to the product of the amplitude of the absolute wavelength of the particle and the fine structure constant ($\lambda\alpha/2\pi$), which we will refer to in this paper as the "integrated wavelength amplitude",

which upon verification turned out to be confirmed.

The equations obtained effectively allow calculating the energy of any localized electromagnetic particles by integrating energy fields mathematically deemed spherically isotropic and whose density radialy decreases from a lower limit of $\lambda\alpha/2\pi$ to an infinite upper limit (∞)

The possibility also came to light that general equations for electric and magnetic fields specific to localized particles could also be established from the same considerations.

By associating quantization of unit charge and integration of the energy associated to the very precisely known dipole moment (the Bohr magneton) and magnetic field of the ground state of the hydrogen atom to the Biot-Savart law, an equation was then developed to calculate the magnetic field of any photon with the absolute wavelength of the photon's energy as the only variable (λ), all other parameters being known constants (π, μ_0, e, c, and α).

From the known equality of density of magnetic and electric energy per unit volume in any electromagnetic field, an equation was then derived from this discrete magnetic field equation to calculate the electric field of any photon with the absolute wavelength of the photon's energy as again the only variable (λ), all other parameters being known constants (π, e, ε_0 and α).

At this point, there remains to be addressed the possibility of relativistic discrete field equations for moving scatterable massive particles, for which the carrying energy must be considered on top of the energy making up the rest mass of such particles.

The natural starting point for such an exploration is the Lorentz equation, which, for straight line motion of a charged particle, provides the only existing equation making use of both **E** and **B** fields to calculate the relativistic velocity of the particle.

By making use of the magnetic field equation previously obtained for photons, that makes use of the absolute wavelength of the particle as the only variable, it is possible to calculate the magnetic field of the electron at rest from its absolute wavelength (the electron Compton wavelength), and to separately calculate the magnetic field of the carrying energy of a moving electron.

From Marmet's demonstration [1], it is clear that the composite magnetic field of an electron in motion can be obtained from the simple sum of the magnetic field of the carrying energy and the magnetic field of the electron at rest.

From relativistic equation ($E=\gamma mc^2$), an equation for relativistic velocity can then be obtained, making use of only the absolute wavelength of the carrying energy and the absolute wavelength of the energy making up the rest mass of the electron.

Having then resolved the **B** element of equation (**E**=v**B**) from only fundamental constants (π, μ_0, e, c, and α) and two absolute wavelengths (λ and λ_c) and the v element from the same two absolute wavelengths (λ and λ_c), a discrete electric field equation can easily be resolved making use of only fundamental constants (π, e, ε_o, and α) and absolute wavelengths (λ and λ_c), which, when used in conjunction with the associated composite **B** field allows calculating the relativistic velocity in straight line of any material particle in motion from only electromagnetic considerations.

These equations support the idea that photons, as well as moving massive particles self-propel at the observed velocity from the interaction of their own internal uniform and orthogonal electric and magnetic fields.

Moreover, in accordance with the only case that allows straight line motion of a charged particle with Lorentz equation, that is, relative E and B values of external uniform fields that verify equation (**E**=v**B**), the new composite discrete field equations for massive moving particles directly explain why moving particles tend to self-propel in straight line, in accordance with Newton's first law; and by similarity, as a limit case with no massive particle involved, (**E**=c**B**) for photons from Maxwell's fourth law provides the same explanations for default straight line motion of photons if no external force is acting on the particle.

Establishing the value of individual electromagnetic fields of electrons, quarks up and quarks down (which are the only scatterable elementary particles known to exist inside atoms) and of their carrying energy inside atoms and nuclei may finally allow determining with precision the contribution of each one of them to the resulting electromagnetic equilibrium inside atoms.

Finally, the fact that these equations support the idea that electromagnetic particles may be self-propelling, directly hints at the possibility that they may exist without the need for underlying fields nor medium of any sort, and that a space geometry that would not impose that energy becomes infinite at r=0 ([3], p. 31) could possibly be conceived of that would allow spherically integrating the energy of scatterable electromagnetic particles from a maximum more realistically compatible with a transverse velocity of energy not exceeding the speed of light, which would be more coherent with the concept of locality.

Energy Calculation by spherical integration

In a recent paper published in the International IFNA-ANS Journal ([1], p. 1 to 7 of the article), Paul Marmet clarified how the magnetic field of an electron in motion increases as the square of its relativistic velocity, that is, in the same proportion as its relativistic mass increases, even though its charge remains constant. When the velocity is small with respect to the speed of light, the following classical equation is obtained (his equation 23), which allows clearly determining the contribution of the magnetic component to the invariant rest mass of the electron.

$$\frac{\mu_0 e^2}{8\pi r_e}\frac{v^2}{c^2} = \frac{m_e}{2}\frac{v^2}{c^2} \tag{1}$$

where $\mathbf{r_e}$ is the Classical electron radius (2.817940285E-15 m), and where **e** is the unit charge of the electron (1.602176462E-19 C).

His starting point was the Biot-Savart equation, in which he quantized the charge in the definition of electrical current and also replaced dt with dx/v, based on the notion that at any given instant, the velocity of current is constant,

$$I = \frac{dQ}{dt} = \frac{d(Ne)}{dt} = \frac{d(Ne)v}{dx} \tag{2}$$

where N represents the number of electrons in one Ampere. By substituting that value of **I** in the scalar version of the Biot-Savart equation,

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi r^2}\sin(\theta)dx \quad , \quad \text{he obtained} \quad d\mathbf{B} = \frac{\mu_0 v}{4\pi r^2}\sin(\theta)\,d(Ne) \tag{3}$$

Without going into the detail of his derivation, which is clearly laid out in his paper, let us only mention that the final stage of his reasoning consists in spherically integrating the electron magnetic energy, whose density is mathematically assumed to be isotropic and deemed to decrease radialy from a minimum distance from r=0 corresponding to r_e to a maximum distance located at infinity (∞).

$$M = \left\{ \frac{\mu_0 e^2 v^2}{2(4\pi)^2 c^2 r^4} \right\} 2\pi \int_0^\pi \sin(\theta) d\theta \int_{r_e}^\infty r^2\, dr \tag{4}$$

In such an integration to infinity, the electron classical radius r_e is the mandatory inferior limit due to the simple fact that integrating any closer to r=0 would accumulate more energy than experimental data warrants. This specific constraint turns out to be the only reason for the existence of that "classical radius" of the electron. After integrating, we finally obtain Marmet's equation no (23), as already mentioned

$$M = \frac{\mu_0 e^2 v^2}{8\pi r_e c^2} = \frac{m_e}{2}\frac{v^2}{c^2} \tag{5}$$

that very precisely corresponds to the total mass of the magnetic field of an electron moving at velocity v, from which he demonstrated that the invariant magnetic field of the electron at rest corresponds to a mass of

$$M = \frac{\mu_0 e^2}{8\pi r_e} = \frac{m_0}{2} \quad , \tag{6}$$

which is exactly half the mass of an electron.

Since this magnetic component represents precisely half of the rest mass of the electron, multiplying it by 2 will of course restitute the electron's total mass, and further multiplying it by c^2 will restitute its total rest energy.

$$\frac{\mu_0 e^2}{8\pi\, r_e} = \frac{m_e}{2} \quad \text{from which} \quad E = m_e c^2 = \frac{\mu_0 e^2 c^2}{4\pi\, r_e} \tag{7}$$

A quick verification will reveal here that multiplying the amplitude of the Compton wavelength, which happens to be the absolute wavelength of the energy making up the mass of an electron (λ_C = c h/E), by the fine structure constant (α) restitutes directly this classical electron radius.

$$r_e = \frac{\lambda_C \alpha}{2\pi} = 2.817940285\ \mathrm{E} - 15\,\mathrm{m} \tag{8}$$

Since setting the lower limit of integration to the integrated amplitude of the Compton wavelength ($\lambda\alpha/2\pi$) in Marmet's equation amounts to spherically integrating the magnetic energy of the particle by treating it mathematically as if it decreased radialy from that lower limit ($\lambda\alpha/2\pi$) to an upper limit located at infinity (∞), the method seemed consequently applicable by definition to any localized electromagnetic particle.

This hinted at the possibility of defining a general equation ([3], p 131), equivalent to E = h*f*, derived from Marmet's equation and this new relation between λ and α, that would allow calculating the energy of any localized photon or even elementary massive particle by spherically integrating its magnetic energy to this presumably universal lower integration limit ($\lambda\alpha/2\pi$), when the upper limit is set to infinity:

$$E = \frac{\mu_0 e^2 c^2}{4\pi r_e} = \frac{\mu_0 e^2 c^2 2\pi}{4\pi \alpha\lambda} = \frac{\mu_0 e^2 c^2}{2\alpha\lambda} \qquad E = \frac{\mu_0 e^2 c^2}{2\alpha\lambda} \tag{9}$$

and alternately, since $\mu_0 = 1/\varepsilon_0 c^2$

$$E = \frac{\mu_0 e^2 c^2}{2\alpha\lambda} = \frac{e^2 c^2}{\varepsilon_0 c^2 2\alpha\lambda} = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \qquad E = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \tag{10}$$

Consequently, we can summarize

$$E = hf = \frac{\mu_0 e^2 c^2}{2\alpha\lambda} = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \tag{11}$$

To confirm the validity of this equation and its perfect harmony with Maxwell's equations, let us now reconcile it with Maxwell's forth equation (Ampere's law generalized) by deriving from it the same equation for calculating the speed of light from both permittivity and permeability constants of vacuum.

$$hf = \frac{\mu_0 e^2 c^2}{2\lambda\alpha} \quad \text{can be written} \quad h\lambda f = \frac{\mu_0 e^2 c^2}{2\alpha} \tag{12}$$

But since $\lambda f = c$, we can reduce to

$$h = \frac{\mu_0 e^2 c}{2\alpha} \quad \text{which then can become} \quad \alpha = \frac{\mu_0 e^2 c}{2h} \tag{13}$$

Now, the standard definition of α, defined from the electrostatic permittivity of vacuum constant, is ([2], p 1.2):

$$\alpha = \frac{e^2}{2\varepsilon_0 hc} \text{, so we can now equate} \quad \frac{\mu_0 e^2 c}{2h} = \frac{e^2}{2\varepsilon_0 hc} \tag{14}$$

Simplifying, we obtain

$$c^2 = \frac{1}{\varepsilon_0 \mu_0} \quad \text{and finally} \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{15}$$

which confirms the soundness of our new equation.

One last point of interest regarding the standard equation defining α, is that it can easily be converted to the electrostatic counterpart of the equation that we just introduced to calculate the energy of a photon from its magnetic component. All that is required is to multiply it term for term by equation $\lambda f = c$:

$$\lambda f\alpha = \frac{e^2 c}{2\varepsilon_0 hc}$$ Isolating λf and simplifying, we effectively obtain $$E = hf = \frac{e^2}{2\varepsilon_0 \lambda\alpha} \qquad (16)$$

Definition of a local discrete magnetic field for isolated photons

It is well documented that while electric force (Coulomb force) on a charged particle does not depend on velocity but that conversely, the magnitude of magnetic force is known to increase with velocity. Or, should we rather say, that the speed of the charged particle is known to increase with magnetic force, while electric charge seems not to vary.

The dependence between velocity and magnetic force is clearly established with the magnetic force equation, that we will apply here to the energy induced in the electron in the ground state of the Bohr atom (we will take the Bohr atom as a handy reference here on account of the well known and documented energy level induced in the hydrogen ground state in this model. Of course, although the related velocity remains somewhat theoretical as far as the hydrogen ground state orbital is concerned, it is nonetheless a quite effective velocity for a free moving electron with the same reference energy):

$$F = qv\mathbf{B} \qquad (17)$$

Where q is the charge of the particle considered, v is its theoretical velocity and **B** is the magnetic field intensity in Tesla. Being a vector product between a *charged particle in motion* seen as a current with a locally active magnetic field, this relation, derived from the Biot-Savart law, wonderfully illustrates the triple orthogonality of electromagnetic energy.

Applied to the isolated hydrogen atom, where electromagnetic equilibrium logically could possibly allow motion of the electron, and knowing that the electrostatic force (F) is directed towards the nucleus as it applies to the electron in motion (ev), which is itself moving perpendicularly to that force, we can much more easily visualize that the magnetic force (**B**), associated to that current (the electron theoretically in motion on the Bohr ground orbit), that is, the spin associated to the electron, can only act perpendicularly to the plane of the orbit, and also of course perpendicularly to the electrostatic force.

Knowing the force at the Bohr radius (8.238721759E-8 N), the charge of the electron, as well as its theoretical classical velocity in the ground state of the Bohr model (2,187,691.252 m/s), it is easy to calculate the magnetic field intensity involved :

$$\mathbf{B}_o = \frac{F_o}{ev} = 235{,}051.7336\text{T} \qquad (18)$$

Knowing besides that $F=mv^2/r$, one can also write

$$ev\mathbf{B}_o = \frac{m_o v^2}{r_o}$$ and finally $$\frac{e}{m_o} = \frac{v}{\mathbf{B}_o r_o} \qquad (19)$$

From the known relation to calculate the electron gyromagnetic moment:

$$\frac{e}{m_o}=\frac{\mu_B}{S_z}\text{ , since } S_z=h/4\pi\text{, we can pose } \frac{e}{m_o}=\frac{4\pi\mu_B}{h} \qquad (20)$$

Which allows us to directly associate the magnetic field intensity at the Bohr radius with the Bohr magneton

$$\frac{v}{\mathbf{B}_o r_o}=\frac{4\pi\mu_B}{h} \qquad (21)$$

and to calculate it from that intensity, since $h=2\pi r_0 m_0 v$ ([3], p 43) :

$$\frac{v}{\mathbf{B}_o r_o}=\frac{4\pi\mu_B}{2\pi r_o m_o v}\text{ and finally } \mu_B=\frac{m_o v^2}{2\mathbf{B}_o}=9.274008988\mathrm{E}-24\,\mathrm{J/T} \qquad (22)$$

Let us note that the electron magnetic dipole moment can also be calculated from the Biot-Savart Law as follows :

$$\mu_B=i\pi a^2=9.274008985\mathrm{E}-24\,\mathrm{J/T} \qquad (23)$$

where i is the current in Coulombs per second, that is, the charge of the electron in Coulomb (e = 1.602176462E-19 C) multiplied by the frequency of the energy at the Bohr orbit (f = 6.579683916E15 Hz), and πa^2 is the surface enclosed in the Bohr orbit, that is, the radius of the orbit (r_o =5.291772083E-11 m) squared and multiplied by π.

So we have determined that the magnetic field at the Bohr orbit is equal to the force at that orbit divided by the charge of the electron and its theoretical velocity

$$\mathbf{B}_o=\frac{F_o}{ev_0} \qquad (24)$$

We also determined that the Bohr magneton is equal to the energy at that orbit divided by $2\mathbf{B}_0$

$$\mu_B=\frac{m_o v^2}{2\mathbf{B}_o}=\frac{E}{2\mathbf{B}_0} \qquad (25)$$

but, μ_B in Joules per Tesla represents by definition the energy at the Bohr radius while B_0 is the intensity of that magnetic field. The magnetic energy at the Bohr orbit will thus be

$$E_m = \mu_B\,\mathbf{B}_0 = 2.179871885\text{ E-18 J} \qquad (26)$$

which corresponds to half of the energy induced at that orbit, by similarity with Marmet's conclusion that magnetic energy constitutes half the mass of the electron. Since $m=E/c^2$, let us see from equation (26) what "mass" corresponds to the magnetic energy induced at the Bohr radius, by applying equation (6) to the Bohr radius magnetic energy:

$$M_m=\frac{E}{c^2}=\frac{\mu_B\mathbf{B}_0}{c^2}=\frac{\mu_0 e^2}{8\pi a_0}=2.425434595\text{ E}-35\,\text{kg} \qquad (27)$$

so we will have

$$\mathbf{B}_0 = \frac{\mu_0 e^2 c^2}{8\pi r_0 \mu_B} \tag{28}$$

But let us recall that applying the quantized charge to the Biot-Savart law reveals that

$$\mu_B = e f \pi r^2 \tag{29}$$

so

$$\mathbf{B}_0 = \frac{\mu_0 e^2 c^2}{8\pi r_0 \mu_B} = \frac{\mu_0 e^2 c^2}{8\pi r_0 e f \pi r_0{}^2} = \frac{\mu_0 e c^2}{8\pi^2 r_0{}^3 f} = 235051.735\mathrm{T} \tag{30}$$

But we also know that the Bohr radius corresponds very precisely to the integrated amplitude of the absolute wavelength of a photon of same energy as that induced at the Bohr radius,

$$r_0 = \frac{\lambda\alpha}{2\pi} \tag{31}$$

we can thus operate the following substitution

$$\mathbf{B} = \frac{\mu_0 e c^2}{8\pi^2 r_0{}^3 f} = \frac{\mu_0 e c^2}{8\pi^2 (\lambda\alpha/2\pi)^3 f} = \frac{\mu_0 e c^2 8\pi^3}{8\pi^2 \lambda^3 \alpha^3 f} = \frac{\mu_0 e c^2 \pi}{\lambda^3 \alpha^3 f} \tag{32}$$

And finally, knowing that the frequency of the energy of a free photon is equal to the speed of light divided by its wavelength, f=c/λ, we can substitute for f

$$\mathbf{B}_0 = \frac{\mu_0 e c^2 \pi}{\lambda^3 \alpha^3 (c/\lambda)} = \frac{\pi \mu_0 e c}{\lambda^2 \alpha^3} = 235051.\ 735\mathrm{T} \tag{33}$$

Which gives us a generalized equation capable of calculating the local magnetic field of any isolated photon from its absolute wavelength, all other parameters being constants

$$\mathbf{B} = \frac{\mu_0 \pi e c}{\alpha^3 \lambda^2} \tag{34}$$

Definition of a local discrete electric field for isolated photons

We know besides, that in an electromagnetic field, the density of magnetic energy per unit volume is equal to the density of electric energy (u_B=u_E)

$$u_B = u_E = \frac{\mathbf{B}^2}{2\mu_0} = \frac{\varepsilon_0 \mathbf{E}^2}{2} \tag{35}$$

so the density of magnetic energy per unit volume at the Bohr orbit would thus be

$$u_B = \frac{\mathbf{B}^2}{2\mu_0} = \frac{(235051735)^2}{2\mu_0} = 2.198300521\text{E}16 \ \ \text{J/m}^3 \qquad (36)$$

The electric field corresponding to that magnetic field would then be

$$\mathbf{E} = \sqrt{\frac{2u_B}{\varepsilon_0}} = 7.04667374\text{E}13 \ \ \text{J/C.m} \qquad (37)$$

On the other hand, by substituting the new definition of **B** in **E**=c**B** detailed in equation (34)

$$\mathbf{E} = c\mathbf{B} = \frac{\pi\mu_0 e c^2}{\alpha^3\lambda^2} \quad \text{and substituting for} \quad \mu_0 = \frac{1}{\varepsilon_0 c^2} \qquad (38)$$

we obtain

$$\mathbf{E} = \frac{\pi e c^2}{\varepsilon_0 c^2 \alpha^3 \lambda^2} = \frac{\pi e}{\varepsilon_0 \alpha^3 \lambda^2} \qquad (39)$$

We have consequently defined a new generalized equation that allows calculating the electric field of any isolated photon founded on the premise that the photon is at all times localized, by integrating spherically its energy, mathematically assumed to be isotropic and to diminish radialy from a minimal distance from its center determined by $\lambda\alpha/2\pi$ to infinity, as previously explained

$$\mathbf{E} = \frac{\pi e}{\varepsilon_0 \alpha^3 \lambda^2} \qquad (40)$$

Now let's see how the values obtainable from this equation compare with the values from more traditional non-local electromagnetism. An easy way to tackle this issue is to assume the presence of n monochromatic photons in the MKS 1 cubic meter reference volume of the electromagnetic energy density equation :

$U = \varepsilon_0 \mathbf{E}^2$ whose units are joules per cubic meter (J/m^3)

If we assume the presence of only one photon in our reference volume, U will of course be equal to the energy of that one photon. Working again with our familiar reference Bohr ground state energy of 27.21138345 eV, that is 4.359743805E-18 J, we can say

$$U = 4.359743805\text{E-}18 \ \text{J/m}^3 \qquad (40a)$$

and of course

$$\mathbf{E} = \sqrt{\frac{U}{\varepsilon_0}} = 7.017075019E-4 \ \text{J/C}\cdot\text{m} \qquad (40b)$$

Let us note that this value mathematically amounts to considering the energy of that single photon as being uniformly spread out within the whole 1 m^3 reference volume, and does not allow localizing the photon with any precision within that volume. Now, let us compare this to the value we found with equation (37), that we can now calculate from the Bohr ground state energy absolute wavelength of (λ = hc/E = 4.556335256E-8 m).

$$\mathbf{E} = \frac{\pi e}{\varepsilon_0 \alpha^3 \lambda^2} = 7.04667374\text{E}13 \text{J/C}\cdot\text{m} \qquad (40c)$$

We can immediately see that (40c) provides a field intensity immensely higher than traditional (40b), which immediately hints that the energy must be much more concentrated and localized than the 1 m^3 reference volume would warrant. We will now proceed to determine what local volume is coherent with this very large (40c) intensity. Let us first calculate the associated energy density

$$U = \varepsilon_0 \mathbf{E}^2 = \varepsilon_0 \left(\frac{\pi e}{\varepsilon_0 \alpha^3 \lambda^2} \right)^2 = \frac{\pi^2 e^2}{\varepsilon_0 \alpha^6 \lambda^4} = 4.396601042\text{E}16 \ \text{J/m}^3 \tag{40d}$$

which confirms an apparent energy density way higher than the traditional non-localized value (40a). Now, the question is, what volume can be associated with such a high local density of energy? We know that U is made up of an energy value in Joules divided by a volume in m^3, so let's see if we can give that form to the equation. Going back to equation (11) defining the energy in joules in the present equation set and comparing it with equation (40d), we observe that equation (11) is a subset of equation (40d), so let us separate the part of (40d) that has the form of an energy in Joules from the rest of the equation.

$$U = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \times \frac{2\pi^2}{\alpha^5 \lambda^3} \tag{40e}$$

The remainder of the equation now has to recognizably take the form of a volume dividing the energy value, so let's proceed:

$$U = E \times \frac{1}{V} = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \times \frac{1}{\left(\dfrac{\alpha^5 \lambda^3}{2\pi^2} \right)} \tag{40f}$$

We can see right off that given that α and π are dimensionless, the units of the parenthesized devisor are correct, that is, cubic meters (m^3), and all that remains to do now is to see if it is proper for calculating a spherical volume. Since the circumference of a sphere is equal to $2\pi r$, we can easily adapt the traditional equation for calculating the volume of a sphere to use the circumference of the sphere which amounts to the wavelength (λ) of the cyclic electromagnetic motion of the energy of our photon, since its amplitude would be $\lambda/2\pi$:

$$V = \frac{4\pi r^3}{3} = \frac{4\pi}{3} \left(\frac{\lambda}{2\pi} \right)^3 = \frac{4\pi}{3} \frac{\lambda^3}{8\pi^3} = \frac{\lambda^3}{6\pi^2} \tag{40g}$$

So, we can see by observing (40g) that we only need to multiply and divide our parenthesized divisor in equation (40f) by mutually canceling values 3 to obtain the required spherical volume equation

$$U = E \times \frac{1}{V} = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \times \frac{1}{3\alpha^5 \left(\dfrac{\lambda^3}{6\pi^2} \right)} \tag{40h}$$

Let us now resolve this equation for our reference energy:

$$U = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \times \frac{1}{3\alpha^5\left(\frac{\lambda^3}{6\pi^2}\right)} = \frac{4.359743805E-18J}{9.916168825E-35m^3} \tag{40i}$$

So we have our exact reference energy in joules divided by the volume that determines the energy density within that volume. Let us calculate the radius of that volume

$$r = \sqrt[3]{\frac{3V}{4\pi}} = 2.871343173E-12m \tag{40j}$$

Now what is the meaning of this radius ? Let us compare it with the amplitude of the wavelength of our reference energy (4.359743805/-18 J) which will be

$$A = \frac{\lambda}{2\pi} = \frac{hc}{2\pi E} = 7.251632784E-9m \tag{40k}$$

and with the lower limit of integration of that photon's energy which is at the origin of the development of the present equation set

$$r_0 = \frac{\lambda\alpha}{2\pi} = \frac{hc\alpha}{2\pi E} = 5.291772086E-11m \tag{40l}$$

So we can observe from comparing the radius (40j) of the spherical volume defined by energy density equation (40h) that this volume is even smaller than the full volume that can be determined by the amplitude of the full wavelength of the photon's energy (40k) and that it is even smaller than the volume that can be determined by the lower limit of spherical integration of its energy (40l).

Consequently, that volume (40h) is definitely coherent with the photon being permanently localized, and localizable at any point along whatever trajectory it may follow.

Note however that this volume does not reflect the actual physical extent of the locally pulsating localized particle, which from all experimental data would be more coherent with the amplitude of the photon's wavelength on either side of the electromagnetic axis of motion normal to the transverse plane of its geometric representation.

This volume (40h) simply is the volume within which the amount of energy of the photon would be contained if it were distributed with uniform density U after being spherically integrated from infinity (∞) to a distance from r=0 corresponding to $\lambda\alpha/2\pi$ as can be extrapolated from Marmet's paper.

It can then be further extrapolated, if the fundamental energy is incompressible, that volume (40h) could be the volume of the photons energy if it were not dynamicaly pulsating. Using this concept to geometrically set it in motion according to a 3 orthogonal spaces geometry [3] would be interesting indeed.

Now back to the **E** and **B** fields equations (40) and (34) of this new equation set. We just saw that energy equation (11) is a subset of the **E** field equation, so from (40) and (34) we can write:

$$\mathbf{E} = \frac{2\pi}{e\alpha^2\lambda}\frac{e^2}{2\varepsilon_0\alpha\lambda} = \frac{2\pi E}{e\alpha^2\lambda} \tag{41}$$

and or course

$$\mathbf{B}=\frac{2\pi}{e c\alpha^2\lambda}\frac{\mu_0 e^2 c^2}{2\alpha\lambda}=\frac{2\pi E}{e c\alpha^2\lambda} \tag{42}$$

Let us verify the soundness of these new generalized equations with a well known energy equal to that induced at the Bohr radius, that is 4.359743805E-18 J, from equation (40), we get

$$\mathbf{E}=\frac{\pi e}{\varepsilon_0\alpha^3\lambda^2}=\frac{2\pi E}{e\alpha^2\lambda}=7.046673731\text{ E13 J/Cm} \tag{43}$$

and from equation (34)

$$\mathbf{B}_0=\frac{\pi\mu_0 e c}{\alpha^3\lambda^2}=\frac{2\pi E}{e c\alpha^2\lambda}=235051.\ 7347\text{ T} \tag{44}$$

If these equations are exact, we should find the speed of light with equation c=**E**/**B**

$$c=\frac{\mathbf{E}}{\mathbf{B}}=\frac{7.046673731\text{ E13}}{235051.7347}=299{,}792{,}458\text{ m/s} \tag{45}$$

which is exact, and will be for any individual localized photon, whatever its energy.

Definition of a general relativistic magnetic field equation for moving massive particles

Our prior use of the energy induced at the Bohr radius to verify some actual figures was not totally innocent. It was also meant to highlight the fact that although an amount of free energy will move at the speed of light, the same amount of energy associated with an electron can move only at the known theoretical velocity associated with the Bohr orbit (2,187,691.252 m/s by classical calculation and 2,187,647.566 m/s by relativistic calculation)

Since from considerations outside the scope of this paper, that carrying energy seems to be of the very same nature as free moving electromagnetic energy, although captive of the electron, we will attempt to see if we can coherently associate the electric and magnetic fields that we just defined for free moving photons to the energy of an electron to confirm the identity.

Let us recall that the equation we just used to calculate the speed of light from the electric and magnetic fields of a photon is derived from Maxwell's 4th equation (Ampere's law generalized).

$$c=\frac{\mathbf{E}}{\mathbf{B}} \tag{46}$$

Let us also put in perspective that the Lorentz equation

$$\mathbf{F}_{(x,t)}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) \tag{47}$$

allows deriving a very similar equation for charged particles in motion, that allows calculating the straight line velocity of an electron from the intensities of constant external orthogonal electric and magnetic fields in which the particle is placed

$$v = \frac{\mathbf{E}}{\mathbf{B}} \tag{48}$$

The condition for straight line motion in this context is precisely that **E**=v**B**, which results in zero net transverse forces being applied to the moving electron, meaning that opposing transverse electric and magnetic resultant forces cancel each other out, causing the particle to move in straight line in the field, a case very familiar in high energy accelerator circles.

Let us now see if it is possible to convert the equation drawn from Maxwell's 4th for a normal photon to that other equation drawn from Lorentz, to calculate the relativistic velocities of an electron by associating the energy of an electron to that of a normal photon, since we postulate here that the energy that determines the velocity of an electron would precisely be that of a perfectly normal photon that would simply be slowed down in relation with the inert energy of the electron that it would be forced to "carry".

One could think in a simplistic way that one only needs to add the energy of the electron fields to those of a photon to obtain the corresponding velocity. Interestingly, this can be done directly for the magnetic energy of the electron and that of the carrier-photon, as Marmet demonstrated ([1], p. 1 to 7).

The magnetic resultant field for a moving electron would then be

$$\mathbf{B} = \frac{\pi\mu_0 ec}{\alpha^3\lambda^2} + \frac{\pi\mu_0 ec}{\alpha^3{\lambda_C}^2}, \quad \text{that is} \quad \mathbf{B} = \frac{\pi\mu_0 ec}{\alpha^3}\frac{\left(\lambda^2 + {\lambda_C}^2\right)}{\lambda^2{\lambda_C}^2} \tag{49}$$

where λ is the absolute wavelength of the carrier-photon (λ = c h/(Energy of the photon)), and λ_C is the Compton wavelength, which is the absolute wavelength of the invariant energy of the electron.

The situation is more complex for the electric field, since from considerations clarified in ([3]), the invariant electron electric energy is apparently oriented orthogonally with respect to the electric energy of the carrier-photon.

The combined electric field of the carrier-photon and electron should thus be a vectorial resultant of a complex product of these electric energies oriented orthogonally with respect to each other. But such a direct calculation could prove extremely difficult in the current state of our comprehension, and we alternately have at our disposal a much simpler method to define the relation, by using the relation equivalent to **E**=c**B**. when dealing with moving massive particles, that is **E**=v**B**.

Redefining gamma

We have just clearly defined the combined magnetic field **B** of the electron in motion thanks to Marmet's contribution, and we now need to establish a clear definition of v, the resolution of both **B** and v ultimately allowing us to clarify the structure of **E** in the case of a moving electron.

We know to start with, that the velocity involved will have to be the relativistic velocity of the particle, so we will start from the well known standard equation for calculating relativistic velocities.

$$E=\gamma mc^2 \quad \text{from which we derive of course} \quad v=c\sqrt{1-\left(\frac{mc^2}{E}\right)^2} \tag{50}$$

We know on the other hand that the value of E used in this equation is made up of the rest energy of the particle plus half of the carrying energy ([3], p. 143), so we can write

$$v=c\sqrt{1-\left(\frac{mc^2}{mc^2+E_P/2}\right)^2} \tag{51}$$

consequently, we can operate the following transformation

$$v=c\sqrt{1-\left(\frac{mc^2}{mc^2+E_P/2}\right)^2}=c\sqrt{1-\frac{1}{\left(\frac{mc^2+E_c/2}{mc^2}\right)^2}}=c\sqrt{1-\frac{1}{\left(1+\frac{1}{mc^2}\frac{E_P}{2}\right)^2}} \tag{52}$$

From the definition of energy clarified in equation (10),

$$E_P=\frac{e^2}{2\varepsilon_0\alpha\lambda}, \quad \text{and} \quad m_0c^2=\frac{e^2}{2\varepsilon_0\alpha\lambda_C} \tag{53}$$

Substituting equations (53) in equation (52)

$$v=c\sqrt{1-\frac{1}{\left(1+\frac{1}{mc^2}\frac{E_P}{2}\right)^2}}=c\sqrt{1-\frac{1}{\left(1+\frac{2\varepsilon_0\alpha\lambda_C}{e^2}\frac{e^2}{4\varepsilon_0\alpha\lambda}\right)^2}}=c\sqrt{1-\frac{1}{\left(1+\frac{\lambda_C}{2\lambda}\right)^2}} \tag{54}$$

Simplifying equation (54) to the limit, we obtain a simplified equation to calculate the relativistic velocity of an electron that uses only one variable, that is, the absolute wavelength of the carrying energy.

$$v=c\sqrt{1-\frac{1}{\left(1+\frac{\lambda_C}{2\lambda}\right)^2}}=\frac{c\sqrt{\lambda_C(4\lambda+\lambda_C)}}{(2\lambda+\lambda_C)} \tag{55}$$

Definition of a general relativistic electric field equation for moving massive particles

So we now have at our disposal clear definitions of both terms located to the right of equation **E**=v**B**. Substituting for v and **B**, we obtain

$$\mathbf{E}=\frac{c\sqrt{\lambda_C(4\lambda+\lambda_C)}}{(2\lambda+\lambda_C)}\frac{\pi\mu_0 ec}{\alpha^3}\frac{(\lambda^2+\lambda_C{}^2)}{\lambda^2\lambda_C{}^2} \tag{56}$$

Substituting for $\mu_0=1/\varepsilon_0 c^2$.

$$\mathbf{E}=\frac{c\sqrt{\lambda_C(4\lambda+\lambda_C)}}{(2\lambda+\lambda_C)}\frac{\pi ec}{\varepsilon_0\alpha^3 c^2}\frac{(\lambda^2+\lambda_C{}^2)}{\lambda^2\lambda_C{}^2} \tag{57}$$

and simplifying, we obtain an electric field equation for the electron in motion whose first part is identical to that of a free photon of same energy as the carrying energy multiplied by the resolved complex ratio of the orthogonal relations of the electrical energy of the electron and the carrier-photon.

$$\mathbf{E}=\frac{\pi e}{\varepsilon_0\alpha^3}\frac{(\lambda^2+\lambda_C{}^2)\sqrt{\lambda_C(4\lambda+\lambda_C)}}{\lambda^2\lambda_C{}^2\quad(2\lambda+\lambda_C)} \tag{58}$$

We now have two equations (49) and (58) for the electric **E** and magnetic **B** fields of a moving electron that requires only one variable, that is, the absolute wavelength of the carrier-photon, just like those that we previously defined for individual photons, two new composite fields derived from those that we defined for individual isolated photons.

Let's confirm by a calculation that these relativistic field equations will provide realistic relativistic velocities. For an energy of 4.359743805E-18 J (27.22 eV), whose absolute wavelength will be $\lambda = ch/E = 4.556335256\text{E-8}$ m, we obtain with equation (58) an electric field of

$$\mathbf{E}=\frac{\pi e}{\varepsilon_0\alpha^3}\frac{(\lambda^2+\lambda_C{}^2)\sqrt{\lambda_C(4\lambda+\lambda_C)}}{\lambda^2\lambda_C{}^2\quad(2\lambda+\lambda_C)}=1.813341121\text{E}13\ \text{J/Cm} \tag{59}$$

and with equation (49) a magnetic field of

$$\mathbf{B}=\frac{\pi\mu_0 ec}{\alpha^3}\frac{(\lambda^2+\lambda_C{}^2)}{\lambda^2\lambda_C{}^2}=8.289000246\ \ \text{E}13\ \ \text{Js/Cm}^2 \tag{60}$$

Resolving the equation

$$v=\frac{\mathbf{E}}{\mathbf{B}}=2{,}187{,}647.566\,\text{m/s} \tag{61}$$

which is precisely the theoretical relativistic velocity corresponding to the Bohr ground state energy.

Calculation with various energies will show that the velocities curve obtained is exactly the same as with the traditional relativistic velocity equation.

Conclusion

These new relativistic field equations, (49) and (58) have been established using the invariant mass of the electron for easier elaboration, but it suffices to replace the Compton wavelength by the absolute wavelength accounting for the rest energy of any other mass we want to consider to generalize the equations to all possible masses.

Now what are the implications of these field equations, (34) and (40) for photons, and (49) and (58) for moving massive particles, that require only the absolute wavelengths of localized electromagnetic events to determine their velocity?

1) That the existence of permanently localized photons and massive particles is directly reconcilable with Maxwell's electromagnetic theory, as hypothesized by Louis de Broglie ([4], p. 277).
2) That it is possible to calculate the individual electromagnetic fields of electrons, quarks up, quarks down and of their carrying energy inside atoms and nuclei and thus determine the contribution of each one of them to the resulting electromagnetic equilibrium inside atoms, even though their potential velocities may be prevented from being expressed due to that equilibrium ([3], Appendix A).
3) That the energy induced in electrons in atoms is electromagnetic in nature, and is of the exact same nature as that of free moving electro-magnetic photons.
4) While Newton's First Law describes the tendency of massive bodies to move in straight line and maintain their state of motion when no outside force is acting on them, these electromagnetic equations for massive particles' motion describe and explain the reason why they behave according to the First Law, whereas the electromagnetic equations established for individual photons explain why the latter also tend to move in straight line when no outside force is acting on them.

Although it allows exact calculation of the energy of discrete localized electromagnetic particles, the mathematical artifact of spherically integrating their energy from infinity seems to be the only reason for the existence of such a lower spherical integration radius as $\lambda\alpha/2\pi$.

It is conceivable that a method will eventually be developed that will allow spherically integrating energy from a maximum more realistically compatible with a transverse velocity of energy not exceeding the speed of light, which would be more coherent with the concept of locality, and would not force arbitrary higher and lower integration limits of ∞ and $\lambda\alpha/2\pi$, which are not obviously coherent with the physical reality of localized particles, possibly by using a space geometry that would not impose that energy becomes infinite at r=0 ([3], p. 31).

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